Simplified Design of Lange Coupler

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***Abstract*—An approximate procedure is described for the computation of characteristic impedances and relative dielectric constants of the odd and even modes on the interdigitated microstrip coupler consisting of N conductors. The procedure requires the knowledge of 1) parameters of the two-conductor coupled microstrip and 2) parameters of the single microstrip.**

THE INTERDIGITATED directional coupler invented by Lange [1] has become a popular element in one-sided microstrip circuits. The design procedure for the Lange coupler has been described recently by Paolino [2]. His procedure requires a numerical solution for the capacitance distribution on a system of N equal conductors, such as shown in Fig. 1(a). The program for such numerical evaluation of capacitances may become quite involved. On the other hand, many of the laboratories working in the design of microwave circuits already have in use a computer program for the solution of the coupled microstrip with $N=2$, such as shown in Fig. 1(b). The best known of such programs has been developed by Bryant and Weiss [3], [4].

Ou [5] originated the idea of using the Bryant–Weiss data in the design of the interdigitated coupler. Ou's method assumes equal velocities for the odd and even modes. This paper will show how to remove this limitation and to obtain approximately both the characteristic impedances and the effective dielectric constants for the even and odd mode. The approximation consists of identifying the mutual and self-capacitances on the system of coupled conductors and evaluating them by the Bryant–Weiss program. The convenience of the described procedure is the use of an existing computer program instead of necessitating the development of a new computer program.

Fig. 1(a) defines the mutual and self-capacitances per unit length on an interdigitated microstrip line with $N=4$ conductors. It is assumed that the three mutual capacitances C_{12} are the same as that of the 2-conductor line with the same spacing s and width w shown in Fig. 1(b). The odd- and even-mode capacitances C_{0o} and C_{0e} of the two-conductor line may be determined by the Bryant–Weiss program. Then, the mutual capacitance C_{12} is given by

$$C_{12}=\frac{C_{0o}-C_{0e}}{2} \quad (1)$$

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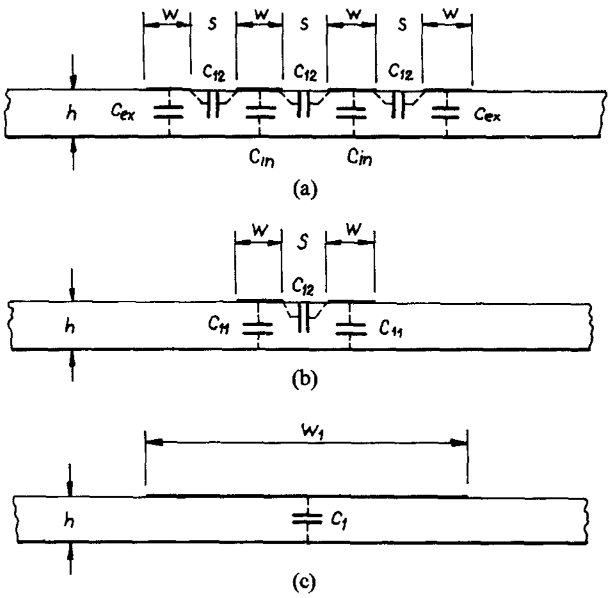

Fig. 1. (a) Microstrip transmission line with $N=4$ conductors, (b) with $N=2$ conductors, and (c) with $N=1$ conductor.

where C_{0o} is the capacitance of the odd mode and C_{0e} is the capacitance of the even mode, as they appear in the printed output of the Bryant–Weiss program.

In Fig. 1(a) the self-capacitances of the two external conductors are denoted by C_{ex}, and the self-capacitances of the two inner conductors by C_{in}. For the purpose of designing the Lange coupler, it is not necessary to know C_{in} and C_{ex} individually because only their sum is needed in computations. For instance, when the 4-conductor line from Fig. 1(a) is driven in the even mode, all conductors have the same potential. The total capacitance to ground for such an excitation can be computed approximately as capacitance C_1 of a single microstripline of the width

$$w_1=4w+3s \quad (2)$$

as shown in Fig. 1(c). The computer program by Bryant–Weiss evaluates C_1 when the input statement is AIR = 0.0. In the Lange coupler, alternate pairs of the conductors are connected together. Therefore, the even-mode capacitance is half the value of total capacitance $(2C_{in}+2C_{ex})$:

$$C_{e4}=C_{in}+C_{ex}\approx\frac{C_1}{2}. \quad (3)$$

By similar reasoning, the odd-mode capacitance C_{o4} of the 4-conductor line is approximately

$$C_{o4}=C_{in}+C_{ex}+6C_{12}\approx\frac{C_1}{2}+3(C_{0o}-C_{0e}). \quad (4)$$

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As seen from (3) and (4), the knowledge of individual capacitances C_{in} and C_{ex} is not needed since only the sum of the two appears in each equation. Analogous approximations may be worked out for an interdigitated line with N conductors, when N is an even number larger than 4.

The assumption is made that the mutual capacitance between any two neighboring conductors is the same as for the two-conductor line and is again computed by (1). In the even mode all N conductors have the same potential, and the total capacitance of such a combination can be evaluated by computing capacitance C_1 of the single microstrip of width

$$w_1 = Nw + (N-1)s. \tag{5}$$

The even-mode capacitance of an N-conductor interdigitated line is then approximately

$$C_{eN} \approx \frac{C_1}{2}. \tag{6}$$

Similarly, the odd-mode capacitance of an N-conductor interdigitated line is given by

$$C_{oN} \approx \frac{C_1}{2} + (N-1)(C_{0o} - C_{0e}). \tag{7}$$

The next step in the procedure requires the knowledge of the capacitances for the same geometries from Figs. 1(a), (b), and (c) but without the dielectric filling. These air-filled capacitances will be denoted by primed symbols, C_1', C_{0e}', and C_{0o}'. Those are available in the computer program by Bryant and Weiss, although they are not explicitly printed in the output. For the single microstrip, the computer output prints the value C_1 and the effective relative dielectric constant $\epsilon_{r,\text{eff}}$ (K-EFF in Bryant–Weiss notation):

$$C_1' = \frac{C_1}{\epsilon_{r,\text{eff}}}. \tag{8}$$

Similarly, the even-mode air-filled capacitance of the 2-conductor microstrip is found from

$$C_{0e}' = \frac{C_{0e}}{\epsilon_{re,\text{eff}}}. \tag{9}$$

The odd-mode air-filled capacitance is

$$C_{0o}' = \frac{C_{0o}}{\epsilon_{ro,\text{eff}}}. \tag{10}$$

Thus the air-filled N-conductor interdigitated line has the even-mode capacitance given by

$$C_{eN}' = \frac{C_1'}{2} \tag{11}$$

and the odd-mode capacitance by

$$C_{oN}' = \frac{C_1'}{2} + (N-1)(C_{0o}' - C_{0e}'). \tag{12}$$

The values computed by (6), (7), (11), and (12) are now used to find the even- and odd-mode characteristic impedances of the N-conductor interdigitated line as follows:

TABLE I
COMPARISON WITH PUBLISHED EXPERIMENTAL DATA

Substrate ϵ_r	w/h	s/h	Measured			Computed	
			Ref.	Z_0(Ω)	C(dB)	Z_0(Ω)	C(dB)
9.6	0.107	0.071	[1]	50.0	3.0	48.1	2.7
9.6	0.112	0.080	[7]	50.0	3.0	47.7	2.8

$$Z_{eN} = \frac{1}{c\sqrt{C_{en} C_{eN}'}} \tag{13}$$

$$Z_{oN} = \frac{1}{c\sqrt{C_{oN} C_{oN}'}} \tag{14}$$

where c is the velocity of light in vacuum.

After the characteristic impedances of the interdigitated structure have been determined, the voltage coupling factor is computed from

$$C = \frac{Z_{eN} - Z_{oN}}{Z_{eN} + Z_{oN}}. \tag{15}$$

The two modes have different propagation velocities. For the quasi-TEM approximation, these velocities are

$$v_{oN} = \frac{c}{\sqrt{\epsilon_{roN}}} \quad v_{eN} = \frac{c}{\sqrt{\epsilon_{reN}}} \tag{16}$$

where the effective relative dielectric constants of both modes are simply equal to the capacitance ratios

$$\epsilon_{roN} = \frac{C_{oN}}{C_{oN}'} \quad \epsilon_{reN} = \frac{C_{eN}}{C_{eN}'}. \tag{17}$$

By knowing the effective dielectric constants, one may compute the response of the coupler over the frequency range of interest and observe its bandwidth and directivity. A simple computer program called CPLR is used for this purpose at the Microwave Laboratory of Iskra, IPT. The necessary expressions are obtained from the well-known quasi-TEM approximation for the coupled lines with inhomogeneous dielectrics, such as those used in [6].

The described procedure has been checked against the published experimental data, in [1], [7] on Lange couplers. The computed values of the loading impedance

$$Z_0 = \sqrt{Z_{oN} Z_{eN}} \tag{18}$$

have been found to depart less than 5 percent from the reported value 50 Ω, and the coupling factor departed less than 0.3 dB from the published results, as shown in Table I.

A number of different values of s/h and w/h has been computed for the substrate $\epsilon_r = 10$ and for $N = 4$ conductors. These data are shown in Fig. 2. Interpolation from this diagram shows that a 3-dB coupler for 50-Ω impedance requires that $w/h = 0.0938$ and $s/h = 0.0883$. For these dimensions, the following values are found:

$$Z_{o4} = 20.41\ \Omega \quad Z_{e4} = 120.07\ \Omega \quad \epsilon_{ro} = 5.41$$
$$\epsilon_{re} = 6.48 \quad Z_0 = 49.5\ \Omega \quad C = -2.98\text{ dB}.$$

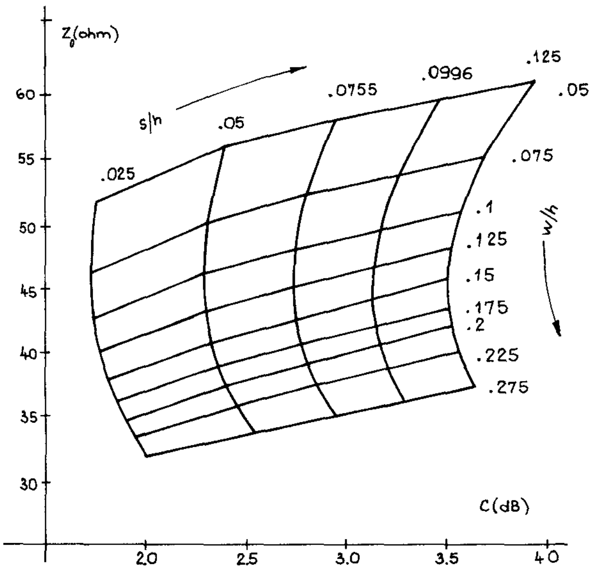

Fig. 2. Approximate design diagram for $\epsilon_r = 10$, $N = 4$.

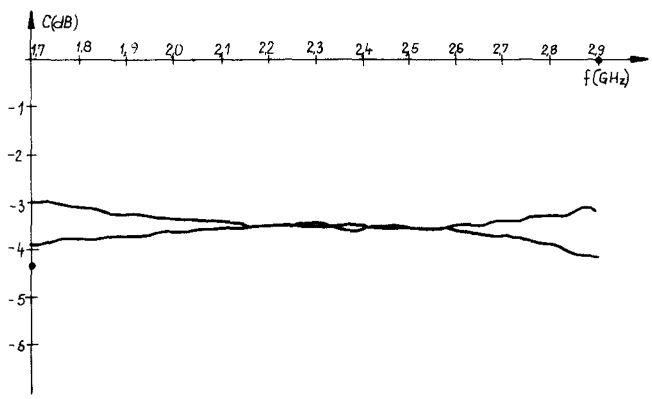

Fig. 3. Measured coupling of the 3-dB Lange coupler with $N = 4$.

The actual dimensions of the fabricated coupler slightly differ from the computed values. Measured under the microscope, the average conductor width was found to be $w/h = 0.101$ and the gap $s/h = 0.0807$. Using these dimensions again in the Bryant–Weiss program (with 20 strips for the gap and 25 strips for the conductor) the following values were computed:

$$Z_{o4} = 19.29\ \Omega \quad Z_{e4} = 119.51\ \Omega \quad \epsilon_{ro} = 5.41$$

$$\epsilon_{re} = 6.49 \quad Z_0 = 48.01\ \Omega \quad C = -2.83\ \text{dB}.$$

The directional coupler, 14.7 mm long, was measured in the frequency range 1.7–2.9 GHz. The measured results, shown in Fig. 3, demonstrate an equal power split between the ports. The average value of coupling is 3.5 dB instead of 3.0 dB. This difference can be attributed to the conductor losses.

The described computational procedure is clearly an approximation. An accurate numerical solution of the entire system of N conductors, such as Paolino's, should give more reliable design data. When such a computational algorithm is available, the dimensions obtained by our approximate procedure could be used as input data.

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